


Neurociencia de Sistemas

- Clase 1. Introducción
- Clase 2. Registros extracelulares y Spike sorting.
- Clase 3. Procesado de información visual.
- Clase 4. Percepción y memoria.
- Clase 5. Decodificación - Teoría de la información.
- Clase 6. Electroencefalografía - Análisis de tiempo-frecuencia y Wavelets.
- Clase 7. Potenciales evocados - Análisis de ensayo único.
- Clase 8. Dinámica no-lineal - Sincronización.

CENTRE FOR
Systems Neuroscience
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Leicester

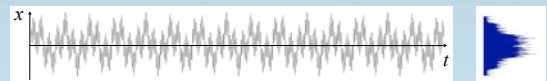


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Slides

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Basic linear analysis



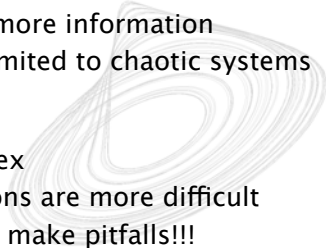
- ✓ Distribution
- ✓ Mean
- ✓ Variance
- ✓ Stationarity

✓ Fourier Transform

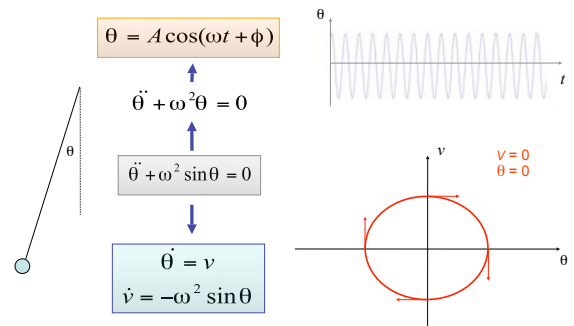
$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt \equiv \langle x(t), e^{-i\omega t} \rangle$$



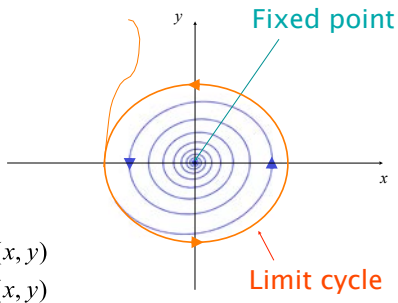
Chaos and non-linear analysis

- ✓ It may give more information
 - ✓ Is not just limited to chaotic systems
 - ✓ It is cool!
 - ✓ More complex
 - ✓ Interpretations are more difficult
 - ✓ Very easy to make pitfalls!!!
- 

Phase space representation

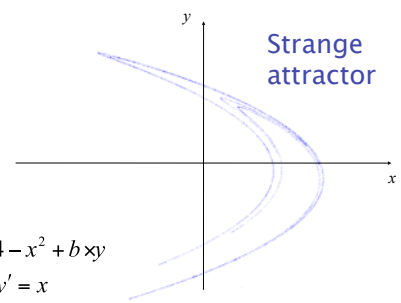


Attractors



$$\begin{aligned}\dot{x} &= f(x, y) \\ \dot{y} &= g(x, y)\end{aligned}$$

Attractors

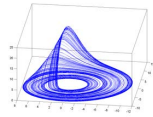


$$\begin{aligned}x' &= 1.4 - x^2 + bxy \\ y' &= x\end{aligned}$$

Toy models

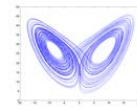
✓ **Rossler**

$$\begin{cases} \dot{x} = y + z \\ \dot{y} = x + 0.2y \\ \dot{z} = 0.2 + z(x - 5.7) \end{cases}$$



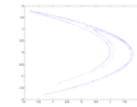
✓ **Lorenz**

$$\begin{cases} \dot{x} = 10(-x + y) \\ \dot{y} = 28x - y - xz \\ \dot{z} = xy - 2.66z \end{cases}$$



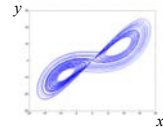
✓ **Henon**

$$\begin{cases} x' = 1.4 - x^2 + bxy \\ y' = x \end{cases}$$

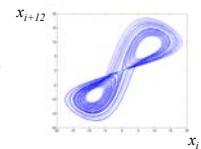


Time delay embedding

$$\begin{cases} \dot{x} = 10(-x + y) \\ \dot{y} = 28x - y - xz \\ \dot{z} = xy - 2.66z \end{cases}$$

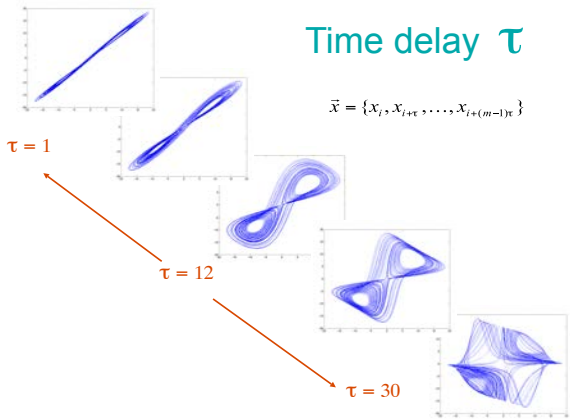


$$\vec{x} = \{x_i, x_{i+\tau}, \dots, x_{i+(m-1)\tau}\}$$



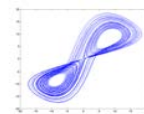
Time delay τ

$$\vec{x} = \{x_i, x_{i+\tau}, \dots, x_{i+(m-1)\tau}\}$$



Embedding dimension m

$$\vec{x} = \{x_i, x_{i+\tau}, \dots, x_{i+(m-1)\tau}\}$$

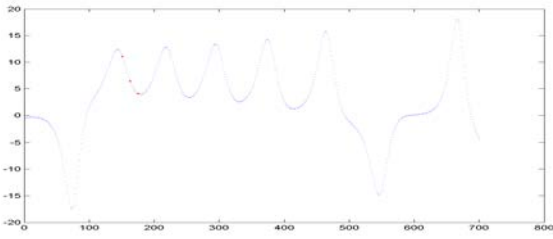


✓ Increase until unfold the attractor

Embedding parameters

$$\vec{x} = \{x_i, x_{i+\tau}, \dots, x_{i+(m-1)\tau}\}$$

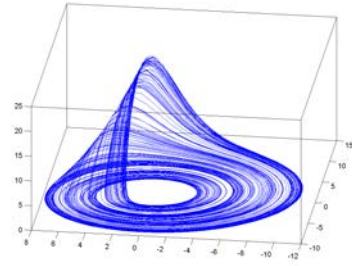
τ : time delay
 m : embedding dimension



✓ Rather than τ or m alone, what matters is $m\tau$

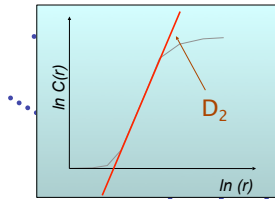
Nonlinear measures

✓ Phase space



Nonlinear measures

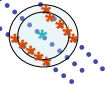
✓ Phase space
 ✓ Neighbors



Correlation dimension

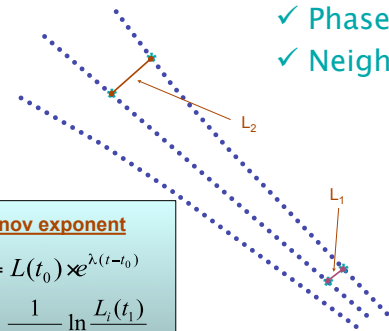
$$C(r) = r^{D_2} \quad \begin{matrix} N \text{ large} \\ r \text{ small} \end{matrix}$$

$$D_2 = \lim_{r \rightarrow 0} \lim_{N \rightarrow \infty} \frac{\ln C(r)}{\ln r}$$



Nonlinear measures

✓ Phase space
 ✓ Neighbors



Lyapunov exponent

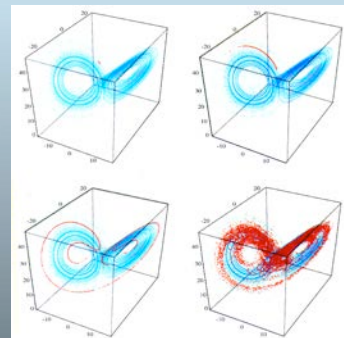
$$L(t) = L(t_0) \times e^{\lambda(t-t_0)}$$

$$\lambda_i = \frac{1}{t_1 - t_0} \ln \frac{L_i(t_1)}{L_i(t_0)}$$

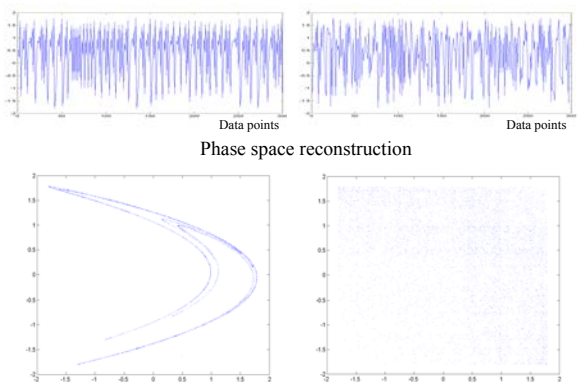
Chaotic attractors

- ✓ Fractal dimension
 - ✓ self-similarity
- ✓ At least 1 positive Lyapunov exponent
 - ✓ sensitivity to initial conditions

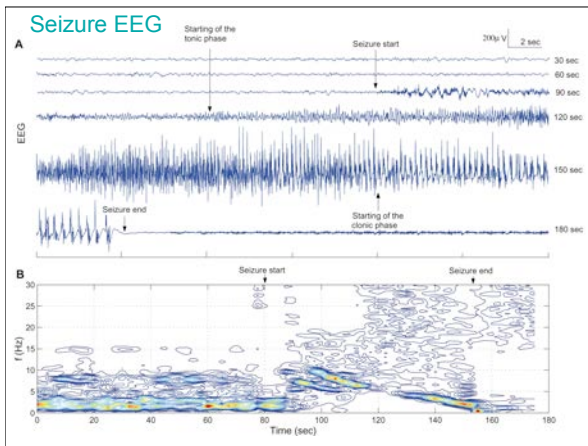
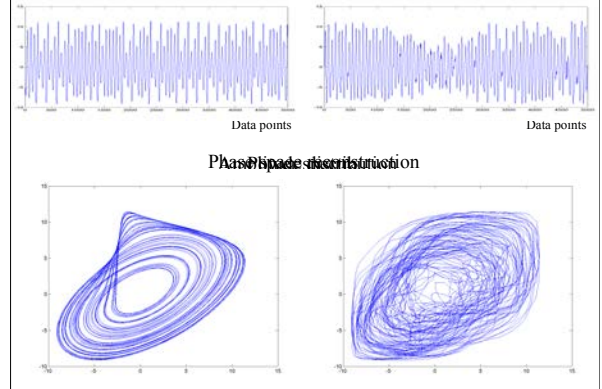
Sensitivity to initial conditions



Are they different?



Are they different?



A personal approach to non-linear analysis...

- ✓ Visual inspection
- ✓ Stationarity
- ✓ Basic linear analysis (Power spectrum)
- ✓ Finding of a good embedding (m, τ)
- ✓ Setting of parameters (e.g. nr. Nearest neighbors)
- ✓ Use of the Non-linear method
- ✓ Does it give more information?
- ✓ Carefull with the interpretation

Synchronization



Why synchronization?

In the EEG:

- Communication between distant brain areas.
- Functional connectivity.
- Quantify different brain states.
- Basic mechanism of epilepsy.

At single cell level:

- Bottom up processes (in vision).
- Alternative coding (instead of firing rates).

Linear measures of synchronization

- Cross-correlation

$$c_{xy}(\tau) = \frac{1}{N-\tau} \sum_i \frac{x_i - \bar{x}}{\sigma_x} \times \frac{y_{i+\tau} - \bar{y}}{\sigma_y}$$

- Coherence

$$C_{xy}(\omega) = (F_x)(\omega) (F_y)^*(\omega)$$

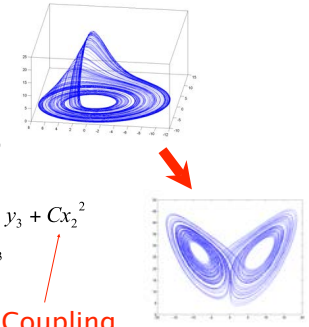
$$\Gamma_{xy}(\omega) = \frac{\langle C_{xy}(\omega) \rangle}{\sqrt{\langle C_{xx}(\omega) \rangle} \sqrt{\langle C_{yy}(\omega) \rangle}}$$

Lorenz driven by a Rossler

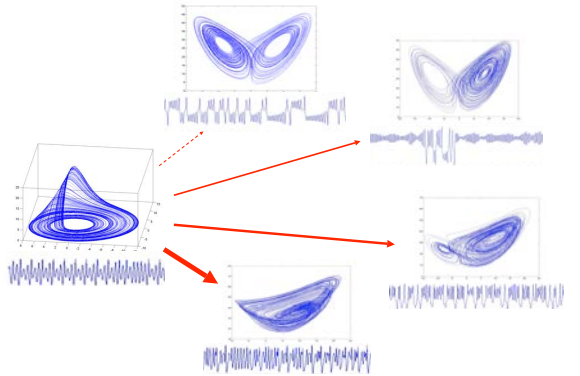
$$\begin{cases} \dot{x}_1 = x_2 + x_3 \\ \dot{x}_2 = x_1 + 0.2x_2 \\ \dot{x}_3 = 0.2 + x_3(x_1 - 5.7) \end{cases}$$

$$\begin{cases} \dot{y}_1 = 10(-y_1 + y_2) \\ \dot{y}_2 = 28y_1 - y_2 - y_1 y_3 + Cx_2^2 \\ \dot{y}_3 = y_1 y_2 - 2.66 y_3 \end{cases}$$

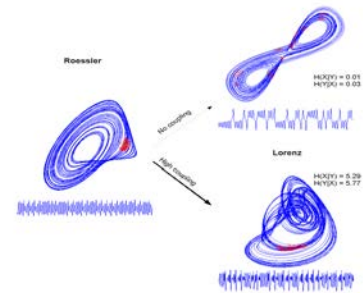
Coupling



Lorenz driven by a Rossler



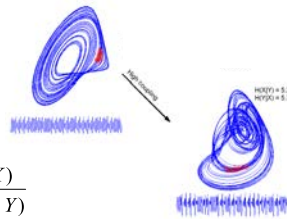
Non-linear interdependencies



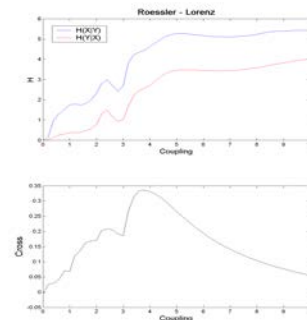
Equations

$$S^{(k)}(X|Y) = \frac{1}{N} \sum_{n=1}^N \frac{R_n^{(k)}(X)}{R_n^{(k)}(X|Y)}$$

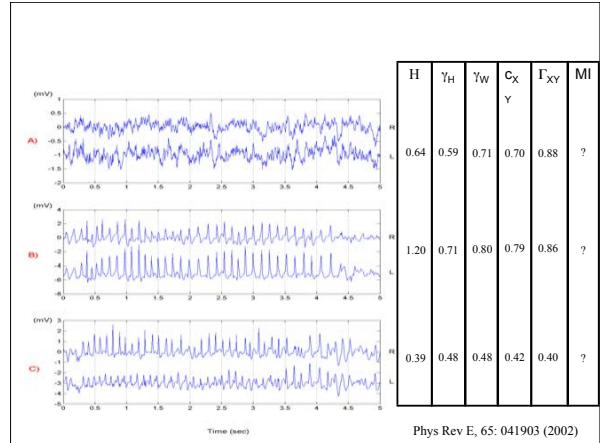
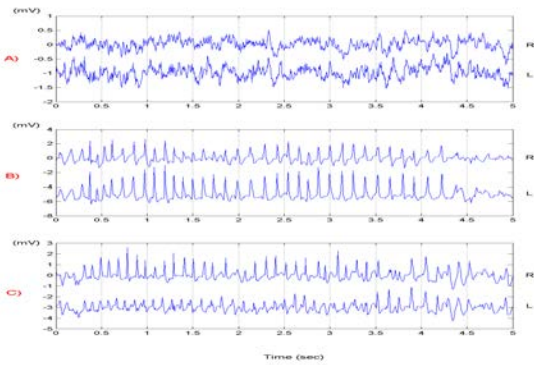
$$H^{(k)}(X|Y) = \frac{1}{N} \sum_{n=1}^N \log \frac{R_n(X)}{R_n^{(k)}(X|Y)}$$



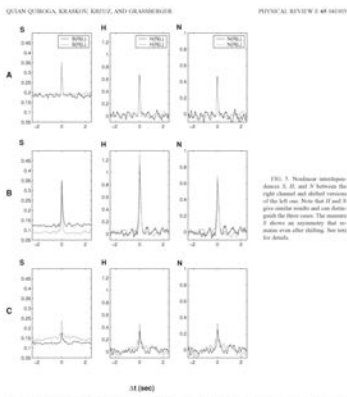
Synchronization of the coupled Roessler – Lorenz systems



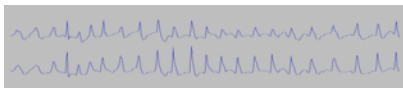
Which one is more synchronized?



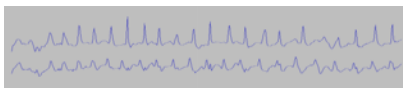
Time-shifted surrogates



Key idea: given 2 signals, look for quasi-synchronous events.

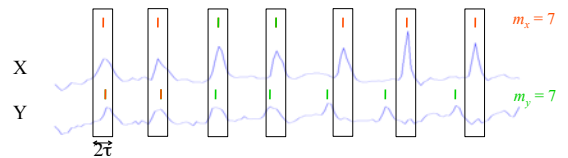


High Synchro



Low Synchro

Outline of the method:

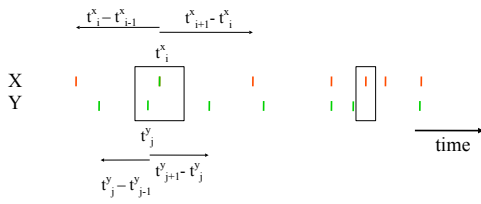


1. Detect events (e.g. local maxima)
2. Draw boxes of length 2τ around each event in X

3. Define: $C^+(Y|X)$: # X_i appears before Y_j
 $C^+(X|Y)$: # Y_j appears before X_i

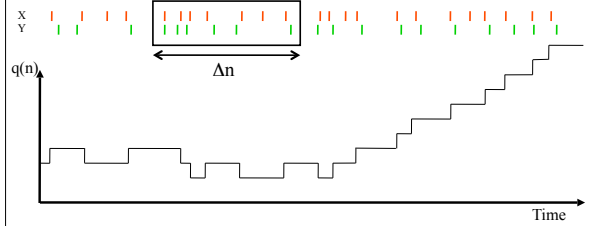
4. Define: $Q_\tau = \frac{C^+(Y|X) + C^+(X|Y)}{\sqrt{m_x m_y}}$ $a_\tau = \frac{C^+(Y|X) - C^+(X|Y)}{\sqrt{m_x m_y}}$
 Synchronization Time asymmetry

Automatic (and local) window τ



$$\tau_{ij} = \min\{t_{i+1}^x - t_i^x; t_i^x - t_{i-1}^x; t_{j+1}^y - t_j^y; t_j^y - t_{j-1}^y\} / 2$$

Time resolved asymmetry: $q(n) = c_n(y|x) - c_n(x|y)$

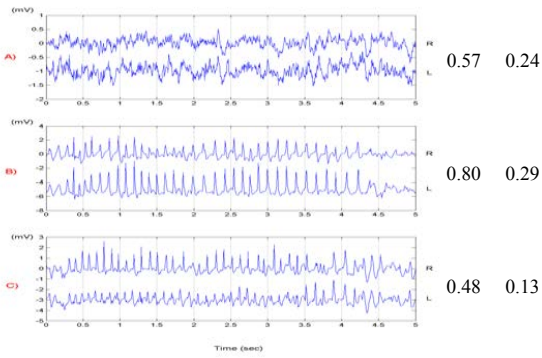


Time resolved synchronization: $Q(n) = c_n(y|x) + c_n(x|y)$

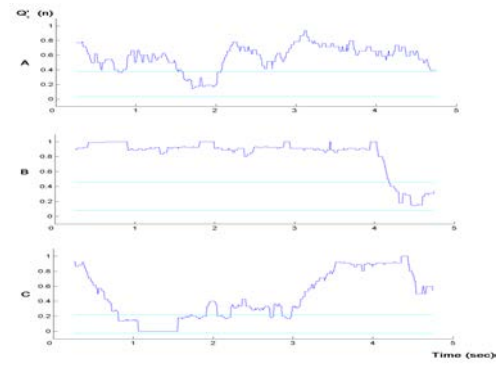
$$Q'(n) = \frac{Q(n) - Q(n - \Delta n)}{\sqrt{\Delta n_x \cdot \Delta n_y}}$$

Application to rat EEGs

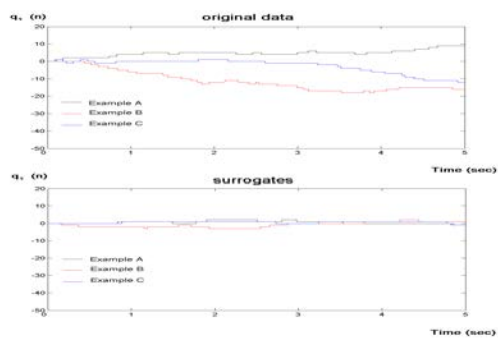
$Q_{\tau=2} \quad Q_{\tau=2}^s$



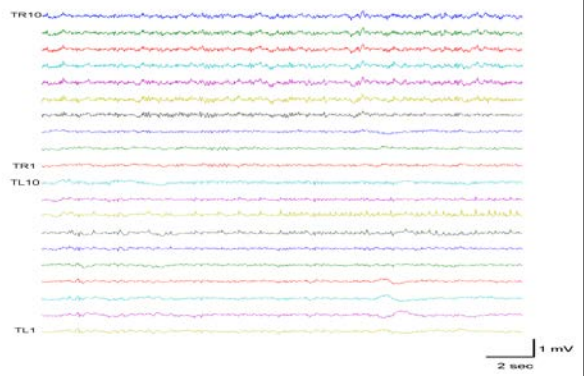
Time resolved synchronization

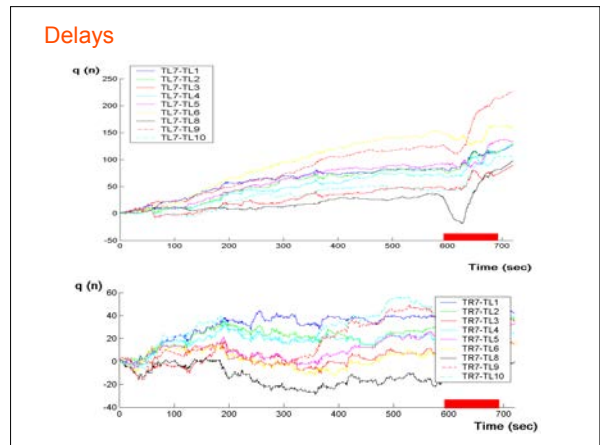
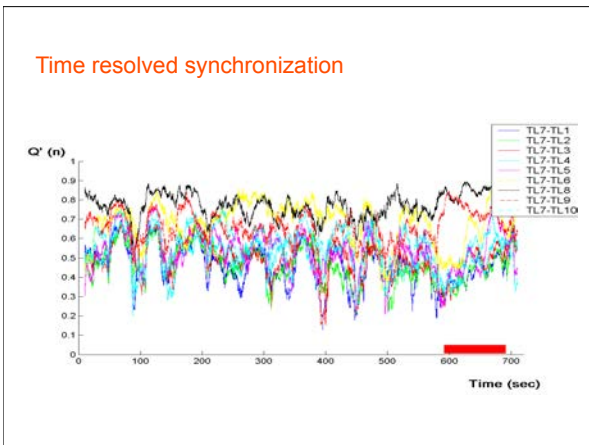
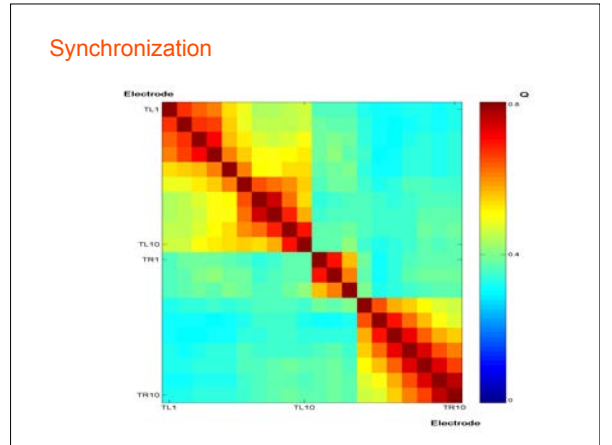
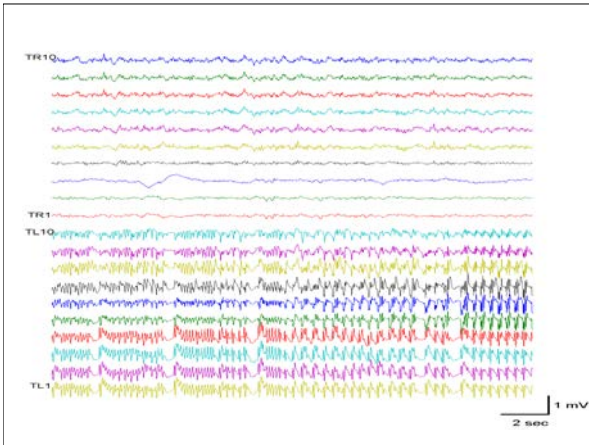
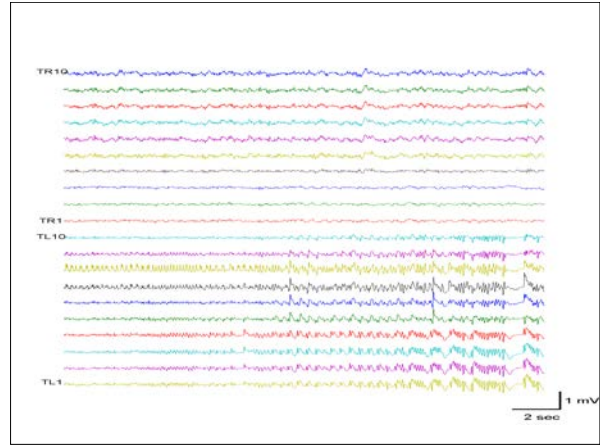
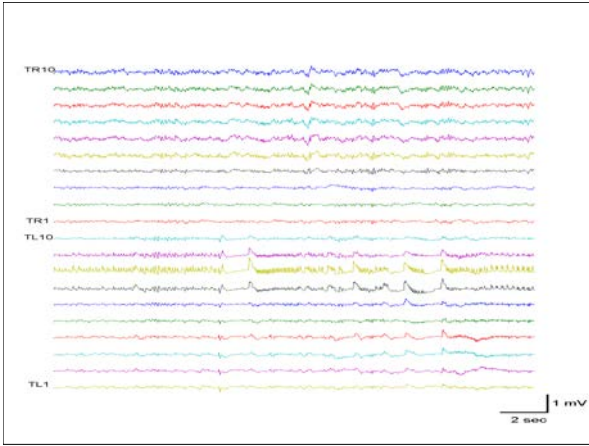


Delay asymmetry



EEG of an epileptic patient





Clase 8. Dinámica no-lineal – Sincronización

Nonlinear multivariate analysis of neurophysiological signals.

Pereda E, Quiñan Quiroga R and Bhattacharya J
Progress in Neurobiology 77: 1-37; 2005.

Performance of different synchronization measures in real data: a case study on electroencephalographic signals.

Quiñan Quiroga R, Kraskov A, Kreuz T and Grassberger P
Phys. Rev. E, 65: 041903; 2002.

Event synchronization: a simple a fast method to measure synchronicity and time delay patterns.

Quiñan Quiroga R, Kreuz T and Grassberger P.
Phys. Rev. E, 66: 041904, 2002.

Learning driver-response relationships from synchronization patterns.

Quiñan Quiroga R, Arnhold J and Grassberger P.
Phys Rev. E, 61: 5142-5148, 2000. (*un ladrillo...*)

Nonlinear Time series analysis. Kantz and Schreiber (*biblia sobre métodos de caos para análisis de datos*)